# TASI 2006 Lectures on Leptogenesis (Very Preliminary)

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# 1 Lecture I: Introduction

## 1.1 Evidence of Baryon number asymmetry

Standard cosmology: Big Bang $\rightarrow$ inflationary expansion (effectively set curvature = 0) $\rightarrow$ expansion continued, yet the expansion rate determined by which component of the Universe density dominated the total energy density

Dark energy $\sim 70\%$

Dart matter $\sim 30\%$

WMAP:

$$\begin{aligned} \Omega_{\text{dark energy}} &= 0.73 \pm 0.04 & (1) \\ \Omega_{\text{matter}} &= 0.27 \pm 0.04 & (2) \end{aligned}$$

where $\Omega \equiv \rho_0/\rho_c$ and $\rho_c$ is the density corresponds to a closed Universe now, $\rho_c = 3H_0^2/8\pi G_N$.

$$\Omega_{\text{matter}} : \begin{cases} \Omega_B = 0.044 \pm 0.004 \\ \Omega_\gamma : \quad \text{neglegible} \end{cases} \tag{3}$$

Thus

$$\frac{\Omega_{\text{DM}}}{\Omega_{\text{matter}}} \sim 85\% \tag{4}$$

$\Rightarrow$ big puzzles:

- nature of dark energy?
- what is dark matter?
- why $\Omega_B$ so small?

<u>Measuring $n_B/n_\gamma \simeq 6 \times 10^{-10}$:</u>

- photon density: directly follow from CMB temperature measurement and from BE statistic

$$T_{now} \simeq 3^0 K \quad \Rightarrow \quad n_\gamma \simeq T_{now}^3 \sim 400/cm^3 \tag{5}$$

- baryon density $n_B \sim 1/m^3$ from:

1. anisotropies in CMB

   $\Omega_B = 0.044$ can be used to infer the ratio of number density of baryons to photons in the Universe, which is measured independently from primodial nucleosynthesis of light elements

2. BBN:

   primodial Deuterium abundance $\leftrightarrow$ agree with WMAP

   $^4He$, $^7Li$ $\leftrightarrow$ discrepancies [may have underestimated errors]

   Deuterium abundance $\Rightarrow$

$$\frac{n_B}{n_\gamma} \equiv \eta_B = (6.1 \pm 0.3) \times 10^{-10} \tag{6}$$

Relate $\eta_B \sim 10^{-10}$ to matter-antimatter asymmetry:

if the Universe is matter-antimatter symmetric at $T \sim 1$ GeV, as the Universe cools further and the inverse process $2\gamma \to B + \overline{B}$ becomes ineffective due to the Boltzmann factor

$\eta_B$ reduces dramatically as a result of the annihilation process: $B + \overline{B} \to 2\gamma$

$$\frac{n_B}{n_\gamma} = \frac{n_{\overline{B}}}{n_\gamma} \simeq 10^{-18} \tag{7}$$

$\Rightarrow$ a primodial matter-anitomatter asymmetry has to exist at $T \sim 1$ GeV

In reality, $\eta_B$ measures

$$\eta_B = \frac{n_B - n_{\overline{B}}}{n_\gamma} = (6.1 \pm 0.3) \times 10^{-10} \tag{8}$$

(more details, see Scott Dodelson's lectures)

### 1.2 Sakharov's three conditions

hypothesis: the observed expanding Universe originated from a superdense initial state with $T_i \sim M_{pl}$. dynamical generation of baryon asymmetry can occur if there exist

- violation of B
- violation of C and CP
- departure from thermal equilibrium

### 1.2.1 Baryon number violation

This condition is obvious since we start from a baryon symmetric universe (B=0) and to evolve it to a universe where $B \neq 0$. Baryon number violation is thus mandatory.

B-violation in GUT:

natural in GUT as quarks and leptons are in the same irrep

it is thus possible to have gauge bosons and scalars mediating interactions among fermions having different B number

B-violation in EW

SM: B and L accidental symmetries, not possible to violate at tree level

t'Hooft 1976: non-perturbative instanton effects may give rise to processes that violate $(B+L)$, but preserve $(B-L)$

classically, B and L are conserved:

$$J_\mu^B = \frac{1}{3}\sum_i \left(\overline{q}_L\gamma_\mu q_L - \overline{u}_L^c\gamma_\mu u_L^c - \overline{d}_L^c\gamma_\mu d_L^c\right) \tag{9}$$

$$J_\mu^L = \sum_i \left(\overline{\ell}_L\gamma_\mu\ell_L - \overline{e}_L^c\gamma_\mu e_L^c\right) \tag{10}$$

$$B = \int d^3x J_0^B(x) \tag{11}$$

$$L = \int d^3x J_0^L(x) \tag{12}$$

at quantum level, B and L are anomalous:

$$\partial_\mu j_B^\mu = \partial_\mu j_L^\mu = n_f\left(\frac{g^2}{32\pi^2}W_{\mu\nu}^a\widetilde{W}^{a\mu\nu} - \frac{g'^2}{32\pi^2}F_{\mu\nu}\widetilde{F}^{\mu\nu}\right) \tag{13}$$

$\Rightarrow$

$(B-L)$ conserved: $\partial^\mu(J_\mu^B - J_\mu^L) = 0$

$(B+L)$ violated due to the vacum structure of non-abelian gauge theories. divergence of the current:

$$\partial^\mu(J_\mu^B + J_\mu^L) = 2n_F\partial_\mu K^\mu \tag{14}$$

change in B and L related to change in topological charges:

$$B(t_f) - B(t_i) = \int_{t_i}^{t_f} dt \int d^3x \partial^\mu J_\mu^B = n_f [N_{cs}(t_f) - N_{cs}(t_i)] \tag{15}$$

$$N_{cs}(t) = \frac{g^3}{96\pi^2} \int d^3x \epsilon_{ijk} \epsilon^{IJK} W^{Ii} W^{Jj} W^{Kk} \tag{16}$$

vacuum to vacuum transition:

$W^{Ii}$: pure gauge configuration

$N_{cs}$: integers $\qquad \Delta N_{cs} = \pm 1,\ \pm 2,\ ... \qquad \Delta B = \Delta L = n_f \Delta N_{cs}$

in SM: $\Delta B = \Delta L = \pm 3$

SU(2) instanton $\Rightarrow$ effective 12 fermion interaction

$$\mathcal{O}_{B+L} = \prod_{i=1,2,3} (q_{L_i} q_{L_i} q_{L_i} L_{L_i}) \tag{17}$$

at T=0, transition rate: $\Gamma \sim e^{-S_{int}} = e^{-4\pi/\alpha} = \mathcal{O}(10^{-165})$, negligible!

In thermal bath: transition can be made not by tunneling but through thermal fluctuation

$$T = 0: \qquad \Gamma = e^{-4\pi/\alpha} = 10^{-165} \tag{18}$$

$$T < T_{EW}: \qquad \Gamma = e^{\frac{-M_W}{\alpha k T}} \tag{19}$$

$$T = T_{EW}: \qquad \Gamma = \alpha T^4 \tag{20}$$

Thus for $T > T_{EW}$, things become very interesting – Baryon number violation is unsuppressed and copious!

### 1.2.2 C and CP violation

A toy model:

$$\mathcal{L} = g_1 X f_2^\dagger f_1 + g_2 X f_4^\dagger f_3 + g_3 Y f_1^\dagger f_3 + g_4 Y f_2^\dagger f_4 + h.c. \tag{21}$$

where $f_{1,..,4}$: fermion states; $X, Y$: heavy scalars

$\mathcal{L}$ leads to the following processes:

$$X \to \overline{f}_1 + f_2,\ \overline{f}_3 + f_4 \tag{22}$$

$$Y \to \overline{f}_3 + f_1,\ \overline{f}_4 + f_2 \tag{23}$$

at tree level:

$$\Gamma(X \to \overline{f}_1 + f_2) = |g_1|^2 I_X = \Gamma(\overline{X} \to f_1 + \overline{f}_2) = |g_1^*|^2 I_{\overline{X}} \tag{24}$$

where the phase space factora $I_X = I_{\overline{X}}$ thus $\Rightarrow \epsilon = 0$

at one-loop:

$$\begin{aligned}
\Gamma(X \to \overline{f}_1 + f_2) &= g_1 g_2^* g_3 g_4^* I_{XY} + c.c. && (25)\\
\Gamma(\overline{X} \to f_1 + \overline{f}_2) &= g_1^* g_2 g_3^* g_4 I_{XY} + c.c. && (26)
\end{aligned}$$

Now $I_{XY}$ includes kinematic factors arising from integrating over the internal momentum loop due to $J$ exchange in $I$ decay

$I_{XY}$ = complex: if $f_{1,..4}$ are allowed to propagate on-shell

Therefore,

$$\Gamma(X \to \overline{f}_1 + f_2) - \Gamma(\overline{X} \to f_1 + \overline{f}_2) = 4iIm(I_{XY})Im(g_1 g_2^* g_3 g_4^*) \tag{27}$$

Total asymmetry due to X and Y decays:

$$\begin{aligned}
\epsilon_X &= \frac{4}{\Gamma_X} Im(I_{XY}) Im(g_1^* g_2 g_3^* g_4)[(B_4 - B_3) - (B_2 - B_1)] && (28)\\
\epsilon_Y &= \frac{4}{\Gamma_Y} Im(I'_{XY}) Im(g_1^* g_2 g_3^* g_4)[(B_2 - B_4) - (B_1 - B_3)] && (29)
\end{aligned}$$

To have non-zero asymmetry, three conditions have to be satisfied:

- two baryon number violating bosons, each of which has mass greater than the sum of the internal loop fermion masses
- C and CP violation arise from interference between 1-loop and tree diagrams, and manifest itself in complex coupling constants
- X, Y have non-degenerate masses

### 1.2.3 Departure from thermal equilibrium

In equilibrium,

$$\begin{aligned}
< B >_T &= Tr(e^{-\beta H} B) = Tr[(CPT)(CPT)^{-1} e^{-\beta H} B)] && (30)\\
&= Tr(e^{-\beta H} (CPT)^{-1} (CPT)] = -Tr(e^{-\beta H} B)
\end{aligned}$$

thus $< B >_T = 0$ in equilibrium $\Rightarrow$ consequence of CPT invariance.

Departure from thermal equilibrium can be achieved by

- out-of-equilibrium decay : GUT Baryogenesis, Leptogenesis
- EW phase transition : EW Baryogenesis
- dynamics of topological defects

**out-of-equilibrium decays:**

necessary non-equilibrium condition provided by expansion of the Universe

when expansion rate is faster than key particle interaction rates $\Rightarrow$ departure from thermal equilibrium can result

in the expanding universe, the initial abundance of $X$ and $\overline{X}$ is thermal: i.e. $n_X = n_{\overline{X}} \sim n_\gamma$

in LTE (local thermal equilibrium),

$$\begin{aligned} n_X &= n_{\overline{X}} &\simeq n_\gamma \quad \text{for} \quad M_X \lesssim T \qquad (31) \\ n_X &= n_{\overline{X}} &\simeq (M_X T)^{3/2} e^{-M_X/T} \ll n_\gamma \quad \text{for} \quad T \lesssim M_X \qquad (32) \end{aligned}$$

when interactions which create and destroy (decay, annihiliation, and their inverse processes) the $X$ and $\overline{X}$ are occurring rapidly on the expansion time scale, i.e. $\Gamma > H \ \Rightarrow$ equilibrium

scale of rates of processes involving $X$ and $\overline{X}$ relative to the expansion rate determined by $M_X$. if $X$ heavy enough, $\Gamma/H$ becomes smaller $\Rightarrow$ less effective

$*$ departure from thermal equilibrium

$$\frac{\Gamma}{H} < 1 \qquad (33)$$

$\Rightarrow$ over abundance of $X$ and $\overline{X}$

precise computation $\Rightarrow$ need to solve Boltzmann equations (more details in Lecture II)

#### 1.2.4 Relating Baryon and Lepton asymmetries

In weakly coupled plasma: can assign a chemical potential $\mu$ to each of the quark, lepton and Higgs field.

In SM: 1 Higgs, $N_f$ generations of fermions $\Rightarrow 5N_f + 1$ chemical potentials:

$$n_i - \overline{n}_i = \frac{1}{6} g T^3 \begin{cases} \beta\mu_i + \mathcal{O}((\beta\mu_i)^3), & \text{fermions} \\ 2\beta\mu_i + \mathcal{O}((\beta\mu_i)^3), & \text{bosons} \end{cases} \qquad (34)$$

Thermal equilibrium of the following processes:

1. Sphaleron process generated by $\mathcal{O}_{B+L}$:

$$\sum_i (3\mu_{q_i} + \mu_{\ell_i}) = 0 \tag{35}$$

2. SU(3) QCD instanton process $\Rightarrow$ interaction between LH and RH quarks, $\prod_i (q_{L_i} q_{L_i} u^c_{R_i} d^c_{R_i})$

$$\sum_i (2\mu_{q_i} - \mu_{u_i} - \mu_{d_i}) = 0 \tag{36}$$

3. at all temperatures, total hypercharge of plasma vanishes:

$$\sum_i (\mu_{q_i} + 2\mu_{u_i} - \mu_{d_i} - \mu_{\ell_i} - \mu_{e_i} + \frac{2}{N_f}\mu_H) = 0 \tag{37}$$

4. require Yukawa and gauge interactions all in equilibrium:

$$\mu_{q_i} - \mu_H - \mu_{d_j} = 0 \tag{38}$$
$$\mu_{q_i} + \mu_H - \mu_{u_j} = 0 \tag{39}$$
$$\mu_{\ell_i} - \mu_H - \mu_{e_j} = 0 \tag{40}$$

For $T = 100$ GeV $\sim 10^{12}$ GeV, which is of interest of baryogenesis, this is the case for gauge interactions. For Yukawa interactions, however, they are in equilibrium only in a more restricted temperature range. But these effects are small, and thus will be neglected in these lectures.

Baryon number density: $n_B = \frac{1}{6} g B T^2$

Lepton number density: $n_L = \frac{1}{6} g L_i T^2$

Baryon number and Lepton number in terms of chemical potentials:

$$B = \sum_i (2\mu_{q_i} + \mu_{u_i} + \mu_{d_i}) \tag{41}$$
$$L = \sum_i L_i \tag{42}$$
$$L_i = 2\mu_{\ell_i} + \mu_{e_i} \tag{43}$$

Impose the equilibrium conditions between different generations, $\mu_{\ell_i} = \mu_\ell$ and $\mu_{q_i} = \mu_q$:

$$\mu_e = \frac{2N_f+3}{6N_f+3}\mu_\ell, \quad \mu_d = -\frac{6N_f+1}{6N_f+3}\mu_\ell, \quad \mu_u = \frac{2N_f-1}{6N_f+3}\mu_\ell \tag{44}$$

$$\mu_q = -\frac{1}{3}\mu_\ell, \quad \mu_H = \frac{4N_f}{6N_f+3}\mu_\ell \tag{45}$$

The corresponding B and L asymmetries:

$$B = -\frac{4}{3}N_f\mu_\ell \tag{46}$$

$$L = \frac{14N_f^2+9N_f}{6N_f+3}\mu_\ell \tag{47}$$

Thus $B$, $L$ and $B-L$ are related by:

$$B = c_s(B-L), \quad L = (c_s-1)(B-L), \quad \text{where} \quad c_s = \frac{8N_F+4}{22N_f+13} \tag{48}$$

For models with $N_H$ Higgses:

$$c_s = \frac{8N_F+4N_H}{22N_f+13N_H} \tag{49}$$

## 1.3 Mechanisms for Baryogenesis and their problems

### 1.3.1 GUT baryongenesis

A single particle physics interaction at high energy (T):

$$G \to H \to ..... \to SU(3)_c \times SU(2)_L \times U(1)_Y \to U(1)_{EM} \tag{50}$$

Examples: SU(5), SO(10), ... (see Kaladi Babu's lectures)

B-violation natural:

- quarks and leptons in same representations
- super heavy gauge bosons mediate B-changing processes

C and CP violation: naturally built into the theory

equilibrium:

- GUTs effective at very early times

- cosmic expansion was much faster then (faster than the interactions of gauge bosons)
- decays are inherently out-of-equilibrium $\Gamma < H$

*Problems:*

- requires high reheating temperature after inflation. can lead to dangerous production of relics – gravitino and moduli problems
- GUT predicts topological remnants (monopoles)
- extremely hard to test experimentally – can't probe the GUT scale using colliders
- EW theory violates baryon number and can erase pre-existing asymmetry

### 1.3.2 EW baryogenesis

departure from thermal equilibrium provided by strong 1st order PT

advantages:

- can be probed in collider experiments

problems: allowed parameter space very small

- require more CPV than provided in SM (may be found in SUSY)
- need strong enough first order phase transition
- in MSSM, this translates into a strong bound on Higgs mass: $m_H \lesssim 120$ GeV
- stop mass needs to be smaller than, or of the order of, top quark mass

### 1.3.3 Affleck-Dine Baryogensis

involve cosmological evolution of scalar fields carrying B-charge

most naturally implemented in SUSY theories

face the same challenges as in GUT Baryogenesis and in EW Baryogenesis

## 1.4 Neutrino Oscillation and Leptonic CP Violation

Sources of CP violation:

- CP violation in CKM matrix of SM

- CP violation in MSSM
- CP violation in lepton sector

### 1.4.1 leptonic CPV

if neutrinos are Majorana particles (which is the case if its small mass is explained by seesaw mechanism), the Majorana condition then forbids the phase redefinition of $N_R$

$\Rightarrow$ additional CP violating phases in lepton sector

*CP violation at high energy:*

consider SM + $\nu_R$:

$$\begin{aligned} \mathcal{L} &= \bar{\ell}_{L_i} i\gamma^\mu \partial_\mu \ell_{L_i} + \bar{e}_{R_i} i\gamma^\mu \partial_\mu e_{R_i} + \overline{N}_{R_i} i\gamma^\mu \partial_\mu N_{R_i} \\ &\quad + f_{ij}\bar{e}_{R_i}\ell_{L_j} H^\dagger + h_{ij}\overline{N}_{R_i}\ell_{L_j} H - \frac{1}{2} M_{ij} N_{R_i} N_{R_j} + h.c. \end{aligned} \tag{51}$$

choose a basis where $f_{ij}$ and $M_{ij}$ are diagonal

The Yukawa matrix $h_{ij}$ in this basis is in general complex

for 3 families: $h$ has 9 phases, out of which, 3 can be absorbed into wave functions of $\ell_{L_i}$ $\Rightarrow$ 6 physical phases

*CPV at low energy*

integrate out the heavy Majorana neutrinos:

$$\begin{aligned} \mathcal{L}_{eff} &= \bar{\ell}_{L_i} i\gamma^\mu \partial_\mu \ell_{L_i} + \bar{e}_{R_i} i\gamma^\mu \partial_\mu e_{R_i} + f_{ii}\bar{e}_{R_i}\ell_{L_i} H^\dagger \\ &\quad + \frac{1}{2}\sum_k h^T_{ik} h_{kj} \ell_{L_i}\ell_{L_j} \frac{H^2}{M_k} + h.c. \end{aligned} \tag{52}$$

$\Rightarrow \quad -\frac{1}{2} M_{\nu_{ij}} \ell_{L_i}\ell_{L_j} \frac{H^2}{<H>^2}$

Majorana mass matrix symmetric:

$\Rightarrow \quad M_{\nu_{ij}}$ has 6 complex independent elements

$\Rightarrow \quad$ 6 phases: 6 - 3 = 3 physical phases

$$U_{MNS} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13}e^{i\delta} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13}e^{i\delta} \end{pmatrix} \tag{53}$$

$$\cdot \begin{pmatrix} 1 & & \\ & e^{i\alpha_{21}/2} & \\ & & e^{i\alpha_{31}/2} \end{pmatrix}$$

the Dirac phase $\delta$:

$$\begin{aligned} P(\nu_\alpha \to \nu_\beta) &= \delta_{\alpha\beta} - 4\sum_{i>j} Re(U_{\alpha i}U_{\beta j}U^*_{\alpha j}U^*_{\beta i}) \sin^2(\Delta m^2_{ij}\frac{L}{4E}) \\ &\quad +2\sum_{i>j} J_{CP} \sin^2(\Delta m^2_{ij}\frac{L}{4E}) \end{aligned} \tag{54}$$

where

$$J_{CP} = -\frac{Im(H_{12}H_{23}H_{31})}{\Delta m^2_{21}\Delta m^2_{32}\Delta m^2_{31}}, \quad H \equiv (M_\nu^{eff})(M_\nu^{eff})^\dagger \tag{55}$$

Majorana phases $\alpha_{21}$ and $\alpha_{31}$: (see Petr Vogel's lectures)

$$\begin{aligned} |\langle m_{ee}\rangle|^2 &= m_1^2 |U_{e1}|^4 + m_2^2 |U_{e2}|^4 + m_3^2 |U_{e3}|^4 + 2m_1m_2 |U_{e1}|^2 |U_{e2}|^2 \cos\alpha_{21} \\ &\quad +2m_1m_3 |U_{e1}|^2 |U_{e3}|^2 \cos\alpha_{31} + 2m_2m_3 |U_{e2}|^2 |U_{e3}|^2 \cos(\alpha_{31}-\alpha_{21}) \end{aligned} \tag{56}$$

Thus only 3 out of the 6 high energy phases are related to low energy observables (will come back to this in Lecture III)

also, we only know how to probe experimentally two of the three low energy phases [See Boris Kayser's lectures]

Leptogenesis interesting as the high energy baryon asymmetry is in principle entirely determined by the neutrino properties.

# 2 Lecture II: standard scenarios

## 2.1 Standard Leptogenesis (Majorana neutrinos)

### 2.1.1 GUT Baryogenesis Revisit

A toy model:

(this subsection closely follow Buchmuller's)

consider heavy particles $X = \overline{X}$ in thermal bath with CP violating and B-violating decays

$$X \to a + b, \quad X \to \overline{a} + \overline{b} \tag{57}$$

with $B(b) = -B(\overline{b}) = -1$ and $B(a) = -B(\overline{a}) = 0$

Suppose $a$ and $b$ are massless and in thermal equilibrium in a plasma with a large number of degrees of freedom, i.e. $g_* \gg 1$

Assume that at some temperature $T_0 > M_X$, $X$ is not in thermal equilibrium. But, due to processes at high T, one has

$$n_X = \frac{g_X}{2} n_\gamma \tag{58}$$

where $n_\gamma$ is the number density of the photons.

CP asymmetry of the partial widths is

$$\Gamma(X \to a + b) = \frac{1}{2}(1+\epsilon)\Gamma, \quad \Gamma(X \to \overline{a} + \overline{b}) = \frac{1}{2}(1-\epsilon)\Gamma \tag{59}$$

with $\epsilon \ll 1$.

Since $X$ is out of equilibrium at $T_0 > M_X$, it cannot follow the exponential drop of the equilibrium distribution $n_X^{eq}$ at $T \sim M_X$: (i.e. over abundance)

Baryon asymmetry generated in $X$ decays:

$$\frac{n_B - n_{\overline{B}}}{n_\gamma} = \epsilon \frac{n_X}{n_\gamma} = \frac{g_X}{2}\epsilon \tag{60}$$

where the change in temperature due to X-decays has been neglected.

interactions with thermal bath:

- decays

$$X \to a + b, \quad X \to \overline{a} + \overline{b} \tag{61}$$

- inverse decays

$$a + b \to X, \quad \overline{a} + \overline{b} \to X \tag{62}$$

In thermal equilibrium, number densities should not change, in particular no baryon asymmetry should be generated. check:

$$\begin{aligned} \dot{n}_X + 3Hn_X &= -\gamma^{eq}(X \to a+b) + \gamma^{eq}(a+b \to X) \\ &\quad -\gamma^{eq}(X \to \overline{a}+\overline{b}) + \gamma^{eq}(\overline{a}+\overline{b} \to X) \end{aligned} \tag{63}$$

where $\gamma^{eq}$ is the reaction density, # of reactions/time $\times$ volume

CPT invariance:

$$\gamma^{eq}(\overline{a}+\overline{b} \to X) = \gamma^{eq}(X \to a+b) \tag{64}$$

$$\gamma^{eq}(a+b \to X) = \gamma^{eq}(X \to \overline{a}+\overline{b}) \tag{65}$$

$\Rightarrow \quad \dot{n}_X + 3Hn_X = 0$

Baryon density:

$$\begin{aligned} \dot{n}_X + 3Hn_X &= \gamma^{eq}(X \to a+b) - \gamma^{eq}(a+b \to X) \\ &= \gamma^{eq}(X \to a+b) - \gamma^{eq}(X \to \overline{a}+\overline{b}) \\ &\propto \frac{1}{2}(1+\epsilon) - \frac{1}{2}(1-\epsilon) \\ &= \epsilon \neq 0 \ (?) \end{aligned} \tag{66}$$

Consider $2 \to 2$ processes: $a + b \to \overline{a} + \overline{b}$ etc.

Reaction densities:

$$\gamma(X \to ab) = \int d\Phi_{123} f_X(p_1) |\mathcal{M}(X \to ab)|^2 \tag{67}$$

$$\gamma(ab \to \overline{a}\overline{b}) = \int d\Phi_{1234} f_a(p_1) f_b(p_2) |\mathcal{M}'(ab \to \overline{a}\overline{b})|^2 \tag{68}$$

where

$$d\Phi_{1,\ldots,n} = \frac{d^3p_1}{(2\pi)^3 2E_1} \cdots \frac{d^3p_n}{(2\pi)^3 2E_n} \cdot (2\pi)^4 \delta^4(p_1 + \ldots - p_n) \tag{69}$$

is the phase integration over particles in initial and final states and

$$f_i(p) = exp(-\beta E_i(p)), \quad n_i(p) = g_i \int \frac{d^3p}{(2\pi)^3} f_i(p), \quad i = N,\ \ell,\ H \quad \text{at} \quad T = 1/\beta_i \tag{70}$$

$\mathcal{M}$ and $\mathcal{M}'$: scattering matrix elements of the indicated processes at T = 0

Note:

$$|\mathcal{M}'(ab \to \overline{a}\overline{b})|^2 = |\mathcal{M}(ab \to \overline{ab})|^2 - |\mathcal{M}_{ris}(ab \to \overline{ab})|^2 \tag{71}$$

Unitarity of the S-matrix:

$$\sum_i (|\mathcal{M}(ab \to i)|^2 - |\mathcal{M}(i \to ab)|^2) = 0 \tag{72}$$

where $i$ denotes intermediate states

$i = a'b'$, $\overline{a}'\overline{b}'$, integration over phase space

$$E_{a'} + E_{b'} = E_a + E_b = E \tag{73}$$

Thus

$$\sum_{ab,a'b'} (|\mathcal{M}(ab \to \overline{a}'\overline{b}')|^2 - |\mathcal{M}(\overline{a}'\overline{b}' \to ab)|^2) = 0 \tag{74}$$

change of baryon density:

$$\begin{aligned} \dot{n}_b + 3Hn_b &= \gamma^{eq}(X \to ab) - \gamma^{eq}(ab \to X) \\ &\quad + \gamma^{eq}(\overline{ab} \to ab) - \gamma^{eq}(ab \to \overline{ab}) \end{aligned} \tag{75}$$

From narrow width approximation:

$$\gamma^{eq}(\overline{ab} \to ab) - \gamma^{eq}(ab \to \overline{ab}) = -\epsilon\gamma_0^{eq} \tag{76}$$

where

$$\gamma_0^{eq} = \gamma^{eq}(X \to ab) + \gamma^{eq}(X \to \overline{ab}) \tag{77}$$

Thus contributions from $2 \to 2$ processes cancel those from decays and inverse decays.

Boltzmann equations for non-equilibrium:

$$\dot{n}_X + 3Hn_X = -(\frac{n_X}{n_X^{eq}} - 1)\gamma_0^{eq} \tag{78}$$

$$\dot{n}_B + 3Hn_B = \epsilon\gamma_0^{eq}(\frac{n_X}{n_X^{eq}} - 1) - \frac{1}{2}\gamma_0^{eq}\frac{n_B}{n_B^{eq}} - 2\gamma_{2\to 2}^{eq}\frac{n_b}{n_b^{eq}} \tag{79}$$

Applications:

SU(5) GUTs offer candidates for X: heavy gauge bosons (V) or heavy leptoquarks (S), which have B-non-conserving decays:

$$V \to \quad \bar{\ell}_L u_R^c, \qquad B = -\frac{1}{3}, \quad B - L = \frac{2}{3} \tag{80}$$

$$q_L d_R^c, \qquad B = \frac{2}{3}, \quad B - L = \frac{2}{3} \tag{81}$$

$$S \to \quad \bar{\ell}_L \bar{q}_L, \qquad B = -\frac{1}{3}, \quad B - L = \frac{2}{3} \tag{82}$$

$$q_l q_L, \qquad B = \frac{2}{3}, \quad B - L = \frac{2}{3} \tag{83}$$

Since B-L is conserved, i.e. V and S carry B-L charge, no B-L can be generated dynamically. And due to the sphaleron processes, $\langle B \rangle = \langle B - L \rangle = 0$.

In SO(10) GUTs, B-L is spontaneously broken and particles with $M_X < M_{B-L}$ can generate a B-L asymmetry. For $M_X \sim M_{GUT} \sim 10^{15} GeV$, the CP asymmetry $\epsilon$ is suppressed.

One also has to worry about the large reheating temperature $T \sim M_{GUT}$ after the inflation, the realization of thermal equilibrium, and in SUSY case, the gravitino problem. These difficulties lead to interest in EW baryogenesis.

BUT...

SO(10) GUTs predict the existence of RH neutrinos,

$$\psi(16) = (q_L,\ u_R^c,\ e_R^c,\ d_R^c,\ \ell_L,\ \nu_R^c) \tag{84}$$

For hierarchycal fermion masses, one easily has

$$M_N \ll M_{B-L} \sim M_{GUT} \tag{85}$$

where $N = \nu_R + \nu_R^c$ is a Majorana fermion.

The decays,

$$N \to \ell H, \quad N \to \overline{\ell H} \tag{86}$$

where $H$ is the SU(2) Higgs doublet, can lead to a lepton asymmetry and, after sphaleron processes, to a baryon asymmetry [$X = N$, $b = \ell$, $a = H$ in the toy model]

### 2.1.2 Leptogenesis

most general Lagrangian involving charged leptons and neutrinos:

$$\mathcal{L}_Y = f_{ij} \bar{e}_{R_i} \ell_{L_j} H^\dagger + h_{ij} \bar{\nu}_{R_i} \ell_{L_j} H - \frac{1}{2} M_{ij} \bar{\nu}^c_{R_i} \nu_{R_j} + h.c. \tag{87}$$

$$\langle H \rangle = v, \quad m_e = f v_1, \quad m_D = h v \ll M \tag{88}$$

$\Rightarrow$ light and heavy neutrino masses:

$$\nu \simeq V_\nu^T \nu_L + V_\nu^* \nu_L^c, \quad N \simeq \nu_R + \nu_R^c \tag{89}$$

with masses

$$m_\nu \simeq -V_\nu^T m_D^T \frac{1}{M} m_D V_\nu, \quad m_N \simeq M \tag{90}$$

$T < M$: RH neutrinos can generate a lepton asymmetry by means of out-of-equilibrium decays

Sphaleron processes: $\Delta L \to \Delta B$

### 2.1.3 the asymmetry

decay at tree level: $N_i \to H + \ell_L$

total decay width is

$$\Gamma_{D_i} = \Gamma(N_i \to H + \ell_L) + \Gamma(N_i \to H^\dagger + \ell_L^\dagger) = \frac{1}{8\pi}(hh^\dagger)_{ii} M_i \tag{91}$$

out-of-equilibrium condition:

$$\Gamma_{D_1} < H\Big|_{T=M_1} \tag{92}$$

this leads to the following constraint on the effective light neutrino mass

$$\widetilde{m}_1 = (h_\nu h_\nu^\dagger)_{11} \frac{v_2^2}{M_1} \simeq 4\sqrt{g_*}\frac{v_2^2}{M_{pl}} \frac{\Gamma_{D_1}}{H}\Big|_{T=M_1} < 10^{-3} eV \tag{93}$$

where $g_*$ = number of relativistic degrees of freedom. For SM, $g_* \simeq 106.75$, while for MSSM, $g_* \simeq 228.75$.

heavy neutrinos are not able to follow the rapid change of the equilibrium particle distribution, once the temperature dropped below the mass $M_1$

eventually, heavy neutrinos will decay, and a lepton asymmetry is generated due to the CP asymmetry that arises through the inteference of the tree level and one-loop diagrams:

$$\begin{aligned} \epsilon_1 &= \frac{\Gamma(N_1 \to \ell H) - \Gamma(N_1 \to \overline{\ell H})}{\Gamma(N_1 \to \ell H) + \Gamma(N_1 \to \overline{\ell H})} \\ &\simeq \frac{1}{8\pi}\frac{1}{(h_\nu h_\nu)_{11}} \sum_{i=2,3} Im\Big\{(h_\nu h_\nu^\dagger)_{1i}^2\Big\} \cdot \left[ f\left(\frac{M_i^2}{M_1^2}\right) + g\left(\frac{M_i^2}{M_1^2}\right)\right] \end{aligned} \tag{94}$$

one-loop vertex corrections:

$$f(x) = \sqrt{x}\left[1 - (1+x)\ln\left(\frac{1+x}{x}\right)\right] \tag{95}$$

one-loop self-energy:

$$g(x) = \frac{\sqrt{x}}{1-x} \tag{96}$$

For $M_1 \ll M_2,\ M_3$:

$$\epsilon_1 \simeq -\frac{3}{8\pi}\frac{1}{(h_\nu h_\nu^\dagger)_{11}}\sum_{i=2,3} Im\left\{(h_\nu h_\nu^\dagger)^2_{1i}\right\}\frac{M_1}{M_i} \tag{97}$$

The amount of lepton asymmetry generated is given by,

$$Y_L \equiv \frac{n_L - \overline{n}_L}{s} = \kappa\frac{\epsilon}{g_*} \tag{98}$$

Net lepton number produced per decay,

$$\Delta L = \frac{1}{\Gamma_X}\sum_n [\Gamma(X \to f_n) - \Gamma(\overline{X} \to \overline{f}_n)] \tag{99}$$

Out-of-equilibrium condition:

$$r \equiv \frac{\Gamma_1}{H|_{T=M_1}} = \frac{M_{pl}}{(1.7)(32\pi)\sqrt{g_*}}\frac{(h_\nu h_\nu^\dagger)_{11}}{M_1} < 1 \tag{100}$$

(i) If $r \ll 1$ for $T_D \lesssim M_X$: inverse decay and 2-2 scattering impotent:

$$\frac{\Gamma_{ID}}{H} \sim \left(\frac{M_X}{T}\right)^{3/2} e^{-M_X/T}\cdot r \tag{101}$$

$$\frac{\Gamma_S}{H} \sim \alpha\left(\frac{T}{M_X}\right)^5 \cdot r \tag{102}$$

$\Rightarrow$ inverse decay and scattering can be safely ignored

$\Rightarrow$ $\Delta B$ produced by decays is not destroyed by $-\Delta B$ produced by inverse decays and scatterings at $T \simeq T_D$, $n_X \simeq n_{\overline{X}} \simeq n_\gamma$

$\Rightarrow$ net baryon neumber density produced by out-of-equilibrium decays is

$$n_L = \Delta L \cdot n_X \simeq \Delta L \cdot n_\gamma \tag{103}$$

(ii) For $r \gg 1$:

abundance of X and $\overline{X}$ tracks the equilibrium values

$\Rightarrow$ no departure from thermal equilibrium

$\Rightarrow$ no lepton number may evolve [see sec. 2.1.1 on general GUT baryogenesis discussion]

$$\frac{n_\ell - n_{\overline{\ell}}}{dt} + 3H(n_\ell - n_{\overline{\ell}}) = \Delta\gamma^{eq} = 0 \tag{104}$$

In general, for $1 < r < 10$, there could still be sizable asymmetry. The wash out effects due to inverse decay and lepton number violating scattering processes together with the time evolution of the system is then accounted for by the factor $\kappa$, which is obtained by solving the Bolzmann equations for the system (next section). An approximation is given by [see Kolb and Turner,"The Early Universe"],

$$10^6 \lesssim r : \quad \kappa = (0.1r)^{1/2} e^{-\frac{4}{3}(0.1)^{1/4}} \quad (< 10^{-7}) \tag{105}$$

$$10 \lesssim r \lesssim 10^6 : \qquad \kappa = \tfrac{0.3}{r(\ln r)^{0.8}} \quad (10^{-2} \sim 10^{-7}) \tag{106}$$

$$0 \lesssim r \lesssim 10 : \qquad \kappa = \tfrac{1}{2\sqrt{r^2+9}} \quad (10^{-1} \sim 10^{-2}) \tag{107}$$

The EW sphaleron effects then convert $Y_L$ into $Y_B$:

$$Y_B \equiv \frac{n_B - n_{\overline{B}}}{s} = cY_{B-L} = \frac{c}{c-1} Y_L \tag{108}$$

#### 2.1.4 Boltzmann equations

out-of-equilibrium processes: generally treated by Boltzmann equations

main processes in thermal bath for leptogenesis:

- decay of N: [D]

$$N \to \ell + H, \qquad N \to \overline{\ell} + \overline{H} \tag{109}$$

- inverse decay of $N$: [ID]

$$\ell + H \rightarrow N, \qquad \overline{\ell} + \overline{H} \rightarrow N \tag{110}$$

- 2-2 scattering:
  - $\Delta L = 1$ scattering:

$$N_1 \ell(\overline{\ell}) \leftrightarrow \overline{t}(t) q(\overline{q}) \qquad [H, s], \qquad N_1 t(\overline{t}) \leftrightarrow \overline{\ell}(\ell) q(\overline{q}) \qquad [H, t] \tag{111}$$

  - $\Delta L = 2$:

$$\ell H \leftrightarrow \overline{\ell H} \qquad [H], \quad \ell\ell \leftrightarrow \overline{HH}, \quad \overline{\ell\ell} \leftrightarrow HH \qquad [H, t] \tag{112}$$

Boltzmann equations:

$$\frac{dN_{N_1}}{dz} = -(D+S)(N_{N_1} - N_{N_1}^{eq}) \tag{113}$$

$$\frac{dN_{B-L}}{dz} = -\epsilon_1 D(N_{N_1} - N_{N_1}^{eq}) - W N_{B-L} \tag{114}$$

where

$$(D, S, W) \equiv \frac{(\Gamma_D, \Gamma_S, \Gamma_W)}{Hz}, \quad z = \frac{M_1}{T} \tag{115}$$

$\Gamma_D$: include both decay and inverse decay

$\Gamma_S$: include $\Delta L = 1$ scattering processes

$\Gamma_W$: inverse decay, $\Delta L = 1$, $\Delta L = 2$ scattering

### 2.2 Dirac Leptogenesis

Could Leptogenesis occur without lepton number violation (Dirac neutrinos)?

Majorana masses:

$$m_\nu \sim \frac{v_{EW}^2}{M_{GUT}}, \quad \Delta m_{atm}^2 \rightarrow M_{GUT} \sim 10^{15} GeV \tag{116}$$

Dirac mass from SUSY breaking:

$$m_\nu \sim \frac{m_{soft}}{M_{GUT}} \cdot v_{EW}, \quad \Delta m_{atm}^2 \rightarrow M_{GUT} \sim 10^{16} GeV \quad \text{with} \quad m_{soft} \sim 1\, TeV \tag{117}$$

Recall: Sphaleron effects

- only LH particles coupled to Sphalerons
- change (B+L) but not (B-L)
- Sphaleron effects in equilibrium for $T_{EW} \lesssim T$

"Majorana" leptogenesis:

(i) RH neutrino decays $\Rightarrow \quad \Delta L \neq 0$

(ii) Sphaleron convert $\Delta L$ partially into $\Delta B$

How not to equilibrate $\nu_R$?

Dirac neutrinos: $\mathcal{L} \supset \lambda \bar{\ell}_L \phi \nu_R$

LR conversion involving Dirac Yukawa couplings:

For LR conversion not to be in equilibrium:

$$\Gamma_{LR} \lesssim H, \quad \text{for} \quad T_{EW} \lesssim T \tag{118}$$

Thus for $m_D < 10\ keV \quad \Rightarrow$ condition satisfied

Dirac leptogenesis:

(i) two stores of $\Delta L$ generated: $\Delta L_{\nu_L}$ and $\Delta L_{other}$

(ii) sphaleron convert $\Delta L_{\nu_L}$ (but not $\Delta L_{other}$) into $\Delta B$

(iii) LR equilibration occurs late (at $T \ll T_{EW}$)

A SUSY realization:

Table 1: field content

| | $U(1)_L$ | $U(1)_N$ | $SU(2)_L$ | $U(1)_Y$ |
|---|---|---|---|---|
| N | -1 | +1 | 1 | 0 |
| L | +1 | 0 | 2 | -1/2 |
| $H_u$ | 0 | 0 | 2 | 1/2 |
| $\phi$ | +1 | -1 | 2 | -1/2 |
| $\overline{\phi}$ | -1 | +1 | 2 | 1/2 |
| $\chi$ | 0 | -1 | 1 | 0 |

Superpotential:

$$W \ni \lambda N \phi H_u + h L \overline{\phi} \chi + M_\phi \phi \overline{\phi} \tag{119}$$

Integrate out $\phi$ and $\overline{\phi}$:

$$\lambda h \frac{N H_u L \chi}{M_\phi} \longrightarrow \lambda h \frac{\langle\chi\rangle}{M_\phi} N H_u L \tag{120}$$

### 2.3 Gravitino problem

For leptogenesis to be effective: $M_1 > 2 \times 10^9$ GeV. Thus, the reheating temperature has to be $T_{RH} > 2 \times 10^9$ GeV.

High $T_{RH}$ leads to overproduction of light states (i.e. gravitinos)

(i) If gravitinos are stable (i.e. LSP), WMAP constraint on DM $\Rightarrow$ stringent bound on gluino mass for any gravitino mass $m_{3/2}$ and $T_{RH}$.

(ii) If gravitinos are unstable, it has long lifetime and decays during and after BBN

gravitinos may have three effects on BBN:

1. speeds up cosmic expansion: increase $n/p$ ratio and thus $^4He$ aboundance
2. radiation decay of gravitinos reduces $n_B/n_\gamma$ : $\psi \to \gamma + \tilde{\gamma}$
3. high energy photons emitted in gravitino decays destroy light elements (D, T, $^3He$, $^4He$) through photo-dissociation reactions

Table 2: photo-dissociation reactions

| reaction | threshold (MeV) |
|---|---:|
| $D + \gamma \to n + p$ | 2.225 |
| $T + \gamma \to n + D$ | 6.257 |
| $T + \gamma \to p + n + n$ | 8.482 |
| $^3He + \gamma \to p + D$ | 5.494 |
| $^4He + \gamma \to p + T$ | 19.815 |
| $^4He + \gamma \to n +^3 He$ | 20.578 |
| $^4He + \gamma \to p + n + D$ | 26.072 |

Observational constraints:

$$0.22 < Y_p = (\rho_{^4He}/\rho_B)_p < 0.24 \tag{121}$$

$$(n_D/n_H) > 1.8 \times 10^{-5} \tag{122}$$

$$(\frac{n_D + n_{^3He}}{n_H})_p < 10^{-4} \tag{123}$$

Thermal production of gravitinos governed by Boltzmann equation:

$$\frac{d}{dt}n_{3/2} + 3Hn_{3/2} \simeq \left\langle \sum_{\rm tot} v \right\rangle \cdot n_{\rm light}^2 \tag{124}$$

where

$\sum_{\rm tot} \sim 1/M_{pl}^2$: total cross section determining the rate of production of gravitinos

$n_{\rm light} \sim T^3$: number density of light particles in thermal bath

the most stringent constraint: $(D +^3 He)$ which requires gravitino abundance to be

$$\frac{n_{3/2}}{s} \simeq 10^{-2}\frac{T_{RH}}{M_{\rm Pl}} \leq 10^{-12} \tag{125}$$

Thus

$$T_{RH} < 10^{8-9} \text{ GeV} \tag{126}$$

Upper bounds on reheating temperature:

$$\begin{array}{ll} m_{3/2} \leq 100 \text{ GeV}: & T_R \leq 10^{6-7} \text{ GeV} \\ 100 \text{ GeV} \leq m_{3/2} \leq 1 \text{ TeV}: & T_R \leq 10^{7-9} \text{ GeV} \\ 1 \text{ TeV} \leq m_{3/2} \leq 3 \text{ TeV}: & T_R \leq 10^{9-12} \text{ GeV} \\ 3 \text{ TeV} \leq m_{3/2} \leq 10 \text{ TeV}: & T_R \leq 10^{12} \text{ GeV} \end{array} \tag{127}$$

More recently, it has been shown that, for hadronic decay modes, $\psi \to g + \tilde{g}$, the bounds are even more stringent, $T_R < 10^{6-7}$ GeV.

There is therefore a conflict between generation of sufficient amount of leptogenesis and not overly producing gravitinos

# 3 Lecture III: non-standard scenarios

## 3.1 Resonance leptogenesis

In the limit $M_{N_i} - M_{N_j} \ll M_{N_i}$, the self-energy diagrams dominate:

$$\epsilon_{N_i}^{\text{Self}} = \frac{Im[(h_\nu h_\nu^\dagger)_{ij}]^2}{(h_\nu h_\nu^\dagger)_{ii}(h_\nu h_\nu^\dagger)_{jj}} \left[ \frac{(M_i^2 - M_j^2) M_i \Gamma_{N_j}}{(M_i^2 - M_j^2)^2 + M_i^2 \Gamma_{N_j}^2} \right] \tag{128}$$

When $M_1^2 - M_2^2 \sim \Gamma_{N_2}$, the asymmetry can be enhanced

CP asymmetry of $\mathcal{O}(1)$ possible when

$$M_1 - M_2 \sim \frac{1}{2}\Gamma_{1,2} \tag{129}$$

$$\frac{Im(h_\nu h_\nu^\dagger)_{ij}^2}{(h_\nu h_\nu^\dagger)_{ii}(h_\nu h_\nu^\dagger)_{jj}} \sim 1 \tag{130}$$

Thus the required RH neutrino mass scale can be significantly lower.

## 3.2 Soft leptogenesis

CP violation can arise in two ways:

- CPV in decays $\Rightarrow$ standard leptogenesis
- CPV in mixing $\Rightarrow$ soft leptogenesis

recall in Kaon system: mismatch between CP eigenstates and mass eigenstates

$\Rightarrow$ CPV $\neq 0$

CP eigenstates: $\frac{1}{\sqrt{2}}(|K^0\rangle \pm |\overline{K}^0\rangle)$

time evolution of the systemdescribed by Schroedinger equation:

$$\frac{d}{dt}\begin{pmatrix} K^0 \\ \overline{K}^0 \end{pmatrix} = \mathcal{H}\begin{pmatrix} K^0 \\ \overline{K}^0 \end{pmatrix} \tag{131}$$

where $\mathcal{H} = \mathcal{M} - \frac{i}{2}\mathcal{A}$.

$\mathcal{M}_{12}$: dispersive part of the transition amplitude

$\mathcal{A}_{12}$: absorptive part of the ampplitude

$$
\begin{aligned}
|K_L\rangle &= p|K^0\rangle + q|\overline{K}^0\rangle && (132)\\
|K_S\rangle &= p|K^0\rangle - q|\overline{K}^0\rangle && (133)
\end{aligned}
$$

non-vanishing CPV $\Rightarrow \quad |\frac{p}{q}| \neq 1$

$$
\left(\frac{q}{p}\right)^2 = \left(\frac{2\mathcal{M}_{12}^* - i\mathcal{A}_{12}^*}{2\mathcal{M}_{12} - i\mathcal{A}_{12}}\right) \tag{134}
$$

In soft leptogenesis, the relevant Lagrangian involving lightest RH sneutrino:

$$
-\mathcal{L} = [\frac{1}{2}BM_1\tilde{\nu}_{R_1}\tilde{\nu}_{R_1} + Ay_{1i}\tilde{L}_i\tilde{\nu}_{R_1}H_u + h.c.] + \tilde{m}^2\tilde{\nu}_{R_1}^\dagger\tilde{\nu}_{R_1} \tag{135}
$$

the superpotential that involves the lightest RH Sneutrino:

$$
W = M_1N_1N_1 + y_{1i}L_iN_1H_u \tag{136}
$$

$\Rightarrow$ these give the following interactions and mass terms:

$$
\begin{aligned}
-\mathcal{L}_A &= \tilde{\nu}[M_1y_{1i}^*\tilde{\ell}_i^*H_u^* + y_{1i}\overline{\tilde{H}}_u\ell_L^i + Ay_{1i}\tilde{\ell}_iH_u] + h.c. && (137)\\
-\mathcal{L}_m &= [M_A^2\tilde{\nu}_{R_1}^\dagger\tilde{\nu}_{R_1} + \frac{1}{2}BM_1\tilde{\nu}_{R_1}\tilde{\nu}_{R_1}] + h.c. && (138)
\end{aligned}
$$

mixture of $\tilde{\nu}_{R_1}$ and $\tilde{\nu}_{R_1}^\dagger$ leads to eigenstates with definite masses

$$
M_\pm \simeq M_1\left(1 \pm \frac{|B|}{2M_1}\right) \tag{139}
$$

the time evolution of the system:

$$
\frac{d}{dt}\begin{pmatrix}\tilde{\nu}_{R_1}^\dagger\\ \tilde{\nu}_{R_1}\end{pmatrix} = \mathcal{H}\begin{pmatrix}\tilde{\nu}_{R_1}^\dagger\\ \tilde{\nu}_{R_1}\end{pmatrix}, \quad \mathcal{H} = \mathcal{M} - \frac{i}{2}\mathcal{A} \tag{140}
$$

where

$$
\mathcal{M} = \begin{pmatrix} 1 & \frac{B^*}{2M_1}\\ \frac{B^*}{2M_1} & 1\end{pmatrix}M_1, \quad \mathcal{A} = \begin{pmatrix} 1 & \frac{A^*}{M_1}\\ \frac{A^*}{M_1} & 1\end{pmatrix}\Gamma_1 \tag{141}
$$

physical eigenstates:

$$\tilde{N}_L = p\tilde{\nu}^\dagger_{R_1} + q\tilde{\nu}_{R_1}, \quad \tilde{N}_H = p\tilde{\nu}^\dagger_{R_1} - q\tilde{\nu}_{R_1} \tag{142}$$

where

$$\left(\frac{q}{p}\right)^2 \simeq 1 + Im\left(\frac{2\Gamma_1 A}{BM_1}\right) \quad \text{non-vanishing CPV} \quad \Rightarrow Im\left(\frac{2\Gamma_1 A}{BM_1}\right) \neq 0 \tag{143}$$

Thus total asymmetry

$$\begin{aligned} \epsilon &= \frac{\sum_f \int_0^\infty [\Gamma(\tilde{\nu}_{R_1}, \tilde{\nu}^\dagger_{R_1} \to f) - \Gamma(\tilde{\nu}_{R_1}, \tilde{\nu}^\dagger_{R_1} \to \overline{f})]}{\sum_f \int_0^\infty [\Gamma(\tilde{\nu}_{R_1}, \tilde{\nu}^\dagger_{R_1} \to f) + \Gamma(\tilde{\nu}_{R_1}, \tilde{\nu}^\dagger_{R_1} \to \overline{f})]} && (144) \\ &= \left(\frac{4\Gamma_1 B}{4B^2 + \Gamma_1^2}\right) \cdot \left(\frac{Im(A)}{M_1}\right) && (145) \end{aligned}$$

where the final states $f = (\tilde{L}H),\ (L\tilde{H})$ with L=+1, and $\overline{f} = (\tilde{L}^\dagger H^\dagger),\ (\overline{L}, \overline{\tilde{H}})$ with L=-1.

$$\epsilon = \left(\frac{4\Gamma_1 B}{4B^2 + \Gamma_1^2}\right) \cdot \left(\frac{Im(A)}{M_1}\right) \delta_{B-L} \tag{146}$$

where $\delta_{B-L}$ takes into account the thermal efects due to difference between occupation numbers of bosons and fermions

$$\frac{n_B}{s} \simeq -c d_{\tilde{\nu}_R} \epsilon \kappa \tag{147}$$

$c = \frac{8N_F + 4N_H}{22N_F + 13N_H}$: amount of B-L asymmetry being converted into B asymmetry

$d_{\tilde{\nu}_R} = 45\zeta(3)/(\pi^4 g_*)$: density of lightest sneutrino in equilibrium in units of entropy density

For $\Gamma_1 = 2B$:

$$R \equiv \frac{4\Gamma_1 B}{\Gamma_1^2 + 4B^2} = 1 : \qquad \text{resonance condition} \tag{148}$$

total decay width: $\Gamma_1 = \frac{1}{4\pi}(y_\nu y_\nu^\dagger)_{11} M_1$

### 3.3 Non-thermal leptogenesis

*non-thermal leptogenesis via inflaton decay* inflation $\rightarrow$ solve the horizon and flatness problem $\rightarrow$ accounts for the origin of density fluctuations

assume inflaton decays dominantly into a pair of lightest RH neutrinos

$$\Phi \rightarrow N_1 + N_1, \qquad \Rightarrow \qquad m_\Phi > 2M_1 \tag{149}$$

for simplicity, also assume that the decay modes into $N_{2,3}$ are energetically forbidden

The produced $N_1$ then subsequently decays into $H + \ell_L$ and $H^\dagger + \ell_L^\dagger$

If $T_R < M_1 \quad \Rightarrow$ out-of-equilibrium condition automatically satisfied

CP asymmetry generated by interference of tree level and one-loop diagrams:

$$\epsilon = -\frac{3}{8\pi}\frac{M_1}{\langle H\rangle^2} m_3 \delta_{eff} \tag{150}$$

where

$$\delta_{eff} = \frac{Im\left\{h_{13}^2 + \frac{m_2}{m_3}h_{12}^2 + \frac{m_1}{m_3}h_{11}^2\right\}}{|h_{13}|^2 + |h_{12}|^2 + |h_{11}|^2} \tag{151}$$

Numerically,

$$\epsilon \simeq -2\times 10^{-6}\left(\frac{M_1}{10^{10}GeV}\right)\left(\frac{m_3}{0.05eV}\right)\delta_{eff} \tag{152}$$

The chain decays $\Phi \rightarrow N_1 + N_1$ and $N_1 \rightarrow H + \ell_L$ or $H^\dagger + \ell_L^\dagger$ reheat the Universe producing not only the lepton number asymmetry but also the entropy for the thermal bath

Ratio of lepton number to entropy density after reheating:

$$\frac{n_B}{s} \simeq -\frac{3}{2}\epsilon\frac{T_R}{m_\Phi} \simeq 3\times 10^{-10}\left(\frac{T_R}{10^6 GeV}\right)\left(\frac{M_1}{m_\Phi}\right)\left(\frac{m_3}{0.05eV}\right) \tag{153}$$

assuming $\delta_{eff} = 1$.

## 3.4 Connection between leptogenesis and neutrino oscillation

### 3.4.1 Models with 2 RH neutrinos

To cancel the Witten anomaly $\Rightarrow$ 2 RH neutrinos

$\Rightarrow \qquad 3 \times 2$ seesaw:

$$\mathcal{L} = \frac{1}{2}(N_1\ N_2)\begin{pmatrix} M_1 & 0 \\ 0 & M_2 \end{pmatrix}\begin{pmatrix} N_1 \\ N_2 \end{pmatrix} + (N_1\ N_2)\begin{pmatrix} a & a' & 0 \\ 0 & b & b' \end{pmatrix}\begin{pmatrix} \ell_1 \\ \ell_2 \\ \ell_3 \end{pmatrix} H + h.c. \quad (154)$$

The effective neutrino mass matrix:

$$M_\nu^{eff} = M_{LR} M_{RR}^{-1} M_{LR}^T = \begin{pmatrix} \frac{a^2}{M_1} & \frac{aa'}{M_1} & 0 \\ \frac{aa'}{M_1} & \frac{a'^2}{M_1} + \frac{b^2}{M_2} & \frac{bb'}{M_2} \\ 0 & \frac{bb'}{M_2} & \frac{b^2}{M_2} \end{pmatrix} \quad (155)$$

where $a,\ b\, b'$ are real, and $a' = |a'|e^{i\delta}$.

If $\delta = 0$ and $a' = \sqrt{2}a,\ b = b',\ \frac{a^2}{M_1} \ll \frac{b^2}{M_2}$:

$\Rightarrow \qquad m_{\nu_1} = 0,\ m_{\nu_2} = \frac{2a^2}{M_1},\ m_{\nu_3} = \frac{2b^2}{M_2}$

$$U = \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ -1/2 & 1/2 & 1/\sqrt{2} \\ 1/2 & -1/2 & 1/\sqrt{2} \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{pmatrix}, \qquad \theta \simeq \frac{m_{\nu_2}}{\sqrt{2}m_{\nu_3}} \quad (156)$$

$$B \propto \xi_B = Y^2 a^2 b^2 \sin 2\delta, \qquad \xi_{osc} = -\left(\frac{a^4 b^4}{M_1^3 M_2^3}\right)(2 + Y^2)\xi_B \propto -B \quad (157)$$

$\Rightarrow$ sign of CPV of neutrino oscillation and that in leptogenesis are related

### 3.4.2 Models with spontaneous CP violation (& triplet leptogenesis)

minimal LR model:

gauge group:

$$SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times P \rightarrow SU(3)_c \times SU(2)_L \times U(1)_Y \quad (158)$$
$$Q = T_{3,L} + T_{3,R} + \frac{1}{2}(B - L)$$

Particle content:

- fermions:

$$Q_{i,L} = \begin{pmatrix} u \\ d \end{pmatrix}_{i,L} \sim (1/2, 0, 1/3), \qquad Q_{i,R} = \begin{pmatrix} u \\ d \end{pmatrix}_{i,R} \sim (0, 1/2, 1/3)$$
$$L_{i,L} = \begin{pmatrix} e \\ \nu \end{pmatrix}_{i,L} \sim (1/2, 0, -1), \qquad L_{i,R} = \begin{pmatrix} e \\ \nu \end{pmatrix}_{i,R} \sim (0, 1/2, -1)$$

- scalars:

$$
\begin{aligned}
\Phi &= \begin{pmatrix} \phi_1^0 & \phi_2^+ \\ \phi_1^- & \phi_2^0 \end{pmatrix} \sim (1/2,\ 1/2,\ 0) \\
\Delta_L &= \begin{pmatrix} \Delta_L^+/\sqrt{2} & \Delta_L^{++} \\ \Delta_L^0 & -\Delta_L^+/\sqrt{2} \end{pmatrix} \sim (1,\ 0,\ 2) \\
\Delta_R &= \begin{pmatrix} \Delta_R^+/\sqrt{2} & \Delta_R^{++} \\ \Delta_R^0 & -\Delta_R^+/\sqrt{2} \end{pmatrix} \sim (0,\ 1,\ 2)
\end{aligned}
$$

Under parity:

$$\Psi_L \leftrightarrow \Psi_R, \quad \Delta_L \leftrightarrow \Delta_R, \quad \Phi \leftrightarrow \Phi^\dagger \tag{159}$$

In general:

$$\langle\Phi\rangle = \begin{pmatrix} \kappa e^{i\alpha_\kappa} & 0 \\ 0 & \kappa' e^{i\alpha_{\kappa'}} \end{pmatrix},\ \langle\Delta_L\rangle = \begin{pmatrix} 0 & 0 \\ v_L e^{i\alpha_L} & 0 \end{pmatrix},\ \langle\Delta_R\rangle = \begin{pmatrix} 0 & 0 \\ v_R e^{i\alpha_R} & 0 \end{pmatrix} \tag{160}$$

To get realistic SM gauge boson masses:

$$\kappa^2 + \kappa'^2 \simeq \frac{2M_W^2}{g^2} \simeq (174GeV)^2 \tag{161}$$

Two triplet vev's are related:

$$v_L = \beta\frac{\kappa^2}{v_R} \tag{162}$$

The Lagrangian is invariant under the following unitary transformations,

$$U_L = \begin{pmatrix} e^{i\gamma_L} & 0 \\ 0 & e^{-i\gamma_L} \end{pmatrix}, \qquad U_R = \begin{pmatrix} e^{i\gamma_R} & 0 \\ 0 & e^{-i\gamma_R} \end{pmatrix} \tag{163}$$

Under these unitary transformations, the fermions and scalars transform as,

$$
\begin{aligned}
&\Psi_L \to U_L\Psi_L, \qquad \Psi_R \to U_R\Psi_R \\
&\Phi \to U_R\Phi U_L^\dagger, \quad \Delta_L \to U_L^*\Delta_L U_L^\dagger, \quad \Delta_R \to U_R^\dagger\Delta_R U_R^\dagger
\end{aligned}
$$

Thus the vev transform as

$$\kappa \to \kappa e^{-i(\gamma_L-\gamma_R)},\ \kappa' \to \kappa' e^{i(\gamma_L-\gamma_R)},\ v_L \to v_L e^{-2i\gamma_L},\ v_R \to v_R e^{-2i\gamma_R} \tag{164}$$

Using these unitary transformations, we can rotate away 2 of the 4 phases:

$$\langle\Phi\rangle = \begin{pmatrix} \kappa & 0 \\ 0 & \kappa' e^{i\alpha_{\kappa'}} \end{pmatrix},\ \langle\Delta_L\rangle = \begin{pmatrix} 0 & 0 \\ v_L e^{i\alpha_L} & 0 \end{pmatrix},\ \langle\Delta_R\rangle = \begin{pmatrix} 0 & 0 \\ v_R & 0 \end{pmatrix} \tag{165}$$

Yukawa sector:

- quarks:

$$-\mathcal{L}_q = \overline{Q}_{i,R}(F_{ij}\Phi + G_{ij}\overline{\Phi})Q_{j,L} + h.c. \tag{166}$$

where $\overline{\Phi} = \tau_2 \Phi^* \tau_2$

mass matrix:

$$M_u = F_{ij}\kappa + G_{ij}\kappa' e^{-i\alpha_{\kappa'}},\ M_d = F_{ij}\kappa' e^{i\alpha_{\kappa'}} + G_{ij}\kappa \tag{167}$$

  - all Yukawa couplings real (SCPV)
  - $\alpha_{\kappa'}$ responsible for all CPV in quark sector
  - to suppress FCNC: $\kappa/\kappa' \simeq m_t/m_b \gg 1$

- leptons:

$$-\mathcal{L}_\ell = \overline{L}_{i,R}(P_{ij}\Phi + R_{ij}\overline{\Phi})L_{j,L} + f_{ij}(L^T_{i,L}\Delta_L L_{j,L} + L^T_{i,R}\Delta_R L_{j,R}) + h.c. \tag{168}$$

mass matrix:

$$M_e = P_{ij}\kappa' e^{i\alpha_{\kappa'}} + R_{ij}\kappa \tag{169}$$

$$M_\nu^{\rm Dirac} = P_{ij}\kappa + R_{ij}\kappa' e^{-i\alpha_{\kappa'}}, \quad M_\nu^{LL} = f_{ij} v_L e^{i\alpha_L}, \quad M_\nu^{RR} = f_{ij} v_R \tag{170}$$

Thus

$$M_\nu^{eff} = M_\nu^{II} - M_\nu^{I} = (f e^{i\alpha_L} - \frac{1}{\beta}P^T f^{-1} P)v_L \tag{171}$$

$$\begin{aligned} M_\nu^I &= (M_\nu^{Dirac})^T (M_\nu^{RR})^{-1} (M_\nu^{Dirac}) \\ &= (\kappa P + \kappa' e^{-i\alpha_{\kappa'}} R)^T (v_R f)^{-1} (\kappa P + \kappa' e^{-i\alpha_{\kappa'}} R) \\ &\simeq \frac{v_L}{\beta} P^T f^{-1} P \end{aligned} \tag{172}$$

$$M_\nu^I = v_L e^{i\alpha_L} f \tag{173}$$

$\Rightarrow$ the 3 low energy phases $\delta$, $\alpha_{21}$, $\alpha_{31}$, are function of $\alpha_L$

they appear in

- neutrino oscillations: $J^\ell_{CP} \propto \sin\alpha_L$
- $0\nu 2\beta$ decay
- leptogenesis

Triplet leptogenesis: two ways to generate lepton number asymmetry

1. $N_1 \to \ell + H^\dagger$

$$\epsilon = \frac{\Gamma(N_1 \to \ell + H^\dagger) - \Gamma(N_1 \to \bar{\ell} + H)}{\Gamma(N_1 \to \ell + H^\dagger) + \Gamma(N_1 \to \bar{\ell} + H)} \tag{174}$$

2. $\Delta^* \to \ell + \ell$

$$\epsilon = \frac{\Gamma(\Delta_L^* \to \ell + \ell) - \Gamma(\Delta_L \to \bar{\ell} + \bar{\ell})}{\Gamma(\Delta_L^* \to \ell + \ell) + \Gamma(\Delta_L \to \bar{\ell} + \bar{\ell})} \tag{175}$$

Whether $N_1$ decay dominates or $\Delta_L$ decay dominates depends upon if $N_1$ is heavier or lighter than $\Delta_L$

a natural scenario is that the triplet Higgs is heavier than the lightest RH neutrino

$\Rightarrow$ $N_1$ decay dominates

Two types of diagrams contribute:

(A) those that appear in standard leptogenesis:

$$\epsilon = \frac{3}{16\pi}\left(\frac{M_1}{v^2}\right)\frac{Im\left[M_D(M_\nu^I)^* M_D^T\right]_{11}}{(M_D M_D^\dagger)_{11}} = 0 \tag{176}$$

(B) the new contribution:

$$\epsilon = \frac{3}{16\pi}\left(\frac{M_1}{v^2}\right)\frac{Im\left[M_D(M_\nu^{II})^* M_D^T\right]_{11}}{(M_D M_D^\dagger)_{11}} \propto \sin\alpha_L \tag{177}$$

Results independent of the choice of the unitary transformations

### 3.5 New developments, Open questions

- Previous solutions to Boltzmann equations did not include flavor dependence: it has recently been shown that flavor effects matter if heavy neutrino masses are hierarchical [hep-ph/0605281]
- **A Fundamental problem:** Boltzmann equations used in present calculations: classical treatment, yet include collision terms that are zero-temperature S-matrix elements which involve quantum interference; also, time evolution of the system should be treated quantum mechanically.

$\Rightarrow$ need quantum Boltzmann equations $\Rightarrow$ Closed-Time-Path (**CTP**) formalism $\Rightarrow$ KT's PhD thesis [see Riotto's ICPT lecture: hep-ph/9807454 Sec. 7.2]